Covariant Field Equations and Causal UV–IR Filtering in a Regularized Curvature Framework

Walter Carlos Fabián Echenique*
walterechenique@abc.gob.ar

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Abstract

We present a covariant geometric framework in which Planck-scale curvature saturation and the observed infrared vacuum sector are connected through a causal UV–IR filtering mechanism. The theory assumes the existence of a maximum admissible curvature associated with the Planck scale, replacing ultraviolet singularities by regular finite-curvature cores.

The observable infrared sector is described through an effective UV–IR projection law relating Planck-scale curvature saturation to a cosmological causal scale associated with the observable Universe. Within this picture, the small observed vacuum density emerges as a residual geometric curvature selected by causal accessibility rather than as a fundamental cosmological constant requiring fine tuning.

A local covariant representation of the filtering mechanism is constructed through auxiliary geometric fields, leading to modified field equations that preserve general covariance and satisfy the Bianchi identities. The resulting effective vacuum sector naturally exhibits running-vacuum behavior without introducing additional matter fields or phenomenological coupling parameters.

The framework is interpreted as an effective geometric description of the observable causal sector of spacetime. Within this domain, the ultraviolet vacuum is projected into a residual infrared curvature compatible with the observed magnitude of dark energy. Extensions of the mechanism beyond the observable horizon remain outside the scope of the present work.

1 Introduction

One of the longstanding problems in gravitational physics is the enormous discrepancy between the ultraviolet vacuum energy density expected from quantum field theory and the tiny infrared vacuum sector inferred from cosmological observations.

Conventional approaches typically treat these scales as largely independent. Ultraviolet physics is associated with Planck-scale quantum fluctuations, while the observed cosmological vacuum is described through an infrared parameter whose physical origin remains unclear.

In previous work, we proposed that these two regimes may be connected through a geometric filtering mechanism. The central idea is that a finite ultraviolet curvature saturation scale exists at the Planck length, while the observable Universe is characterized by a finite causal scale associated with the cosmological horizon. The enormous separation between these two scales naturally generates an effective infrared curvature residue.

Within this framework, the observable infrared curvature is described by

$$K_{\text{eff}}(t) = K_P \left(\frac{l_P}{R_H(t)} \right)^2, \tag{1}$$

*ORCID: 0009-0001-7373-9573

where K_P denotes the Planckian curvature saturation scale and $R_H(t)$ represents the cosmological causal scale accessible to observation.

The purpose of the present work is not to introduce a new cosmological component, but rather to formulate a covariant geometric description of this ultraviolet-to-infrared projection mechanism. In particular, we develop a local covariant representation of the filtering process using auxiliary fields and derive the corresponding modified gravitational field equations.

A key aspect of the construction is that the causal horizon is interpreted as an observational and causal scale rather than as a physical boundary of spacetime. The framework assumes a single homogeneous and isotropic spacetime and restricts its physical interpretation to the observable domain accessible through causal interactions.

The resulting formulation connects ultraviolet curvature saturation, causal accessibility, and infrared vacuum observables within a unified geometric framework. This allows the filtering mechanism to be expressed directly at the level of the gravitational field equations while preserving general covariance and consistency with the Bianchi identities.

The paper is organized as follows. Section 2 introduces the ultraviolet curvature saturation scale and the associated regularized vacuum sector. Section 3 develops the causal filtering mechanism and the ultraviolet-to-infrared projection law. Section 4 clarifies the ontological interpretation and the domain of validity of the framework. Sections 5 and 6 construct the covariant formulation and derive the modified field equations. Section 7 analyzes the resulting infrared vacuum sector, while Section 8 discusses implications, limitations, and future directions.

The present work does not attempt to describe the global structure of the entire Universe. Instead, it focuses on the observable causal sector accessible to a given observer. Within this domain, a simple UV–IR projection law is proposed, relating Planck-scale curvature saturation to an effective infrared curvature determined by the cosmological causal scale. The resulting framework should therefore be interpreted as an effective geometric description of the observable vacuum sector rather than as a complete theory of all spacetime regions.

2 Ultraviolet Curvature Saturation

2.1 Planckian Curvature Bound

A fundamental assumption of the present framework, motivated by the existence of a minimal physical length scale, is that spacetime curvature cannot grow without bound.. The existence of a minimal physical length scale, identified with the Planck length l_P, naturally implies the existence of a maximum admissible curvature.

Since curvature has dimensions of inverse length squared, the characteristic ultraviolet curvature scale is determined by

$$K_P = \frac{3}{l_P^2}. \tag{2}$$

This quantity defines the maximum curvature accessible to the spacetime geometry. Beyond this scale, the classical notion of continuously increasing curvature loses physical meaning and is replaced by a saturated geometric regime.

The Planckian curvature bound plays the role of an ultraviolet regulator and provides the fundamental geometric scale from which the infrared sector will later emerge through causal filtering.

2.2 Regularized Geometric Core

The existence of a finite ultraviolet curvature implies that spacetime singularities are replaced by regular geometric cores.

In the limit $r \to 0$, the curvature approaches the finite saturation value

$$K(r) \to K_P. \tag{3}$$

Therefore, the central region of the geometry behaves as a finite-curvature vacuum configuration rather than a singular spacetime point.

The regularized core possesses the properties of a local de Sitter-like region, characterized by a nearly constant curvature and finite energy density. The classical divergences associated with black-hole singularities or cosmological singularities are replaced by a smooth geometric phase governed by the Planck curvature scale.

In this picture, ultraviolet regularization is not imposed externally but emerges from the existence of a maximal admissible curvature.

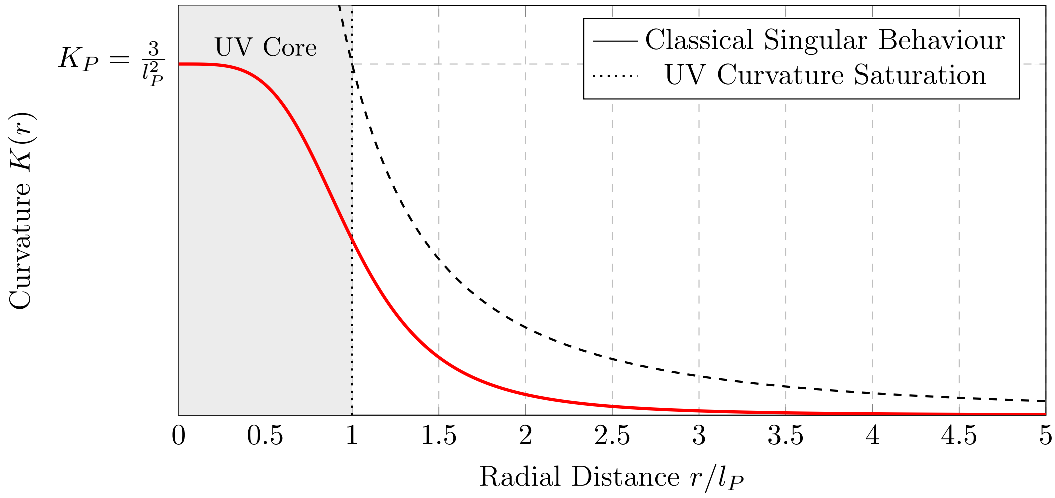

Figure 1: Schematic curvature profile near the ultraviolet regime. In classical General Relativity the curvature diverges as $r \to 0$, generating a spacetime singularity. In the present framework the existence of a Planckian curvature bound $K_P = 3/l_P^2$ replaces the singularity by a finite regularized core. The ultraviolet sector therefore approaches a local de Sitter–like geometry characterized by curvature saturation rather than divergence.

2.3 Ultraviolet Vacuum Sector

The saturated curvature core defines a universal ultraviolet vacuum sector.

Its associated vacuum energy density is obtained through the Einstein curvature-density relation

$$\rho_{\text{UV}} = \frac{c^2}{8\pi G} K_P. \tag{4}$$

Using the Planck curvature bound, one obtains

$$\rho_{\text{UV}} = \frac{3c^2}{8\pi G l_P^2}. \tag{5}$$

This corresponds to the maximum vacuum energy density compatible with the regularized ultraviolet geometry.

The ultraviolet vacuum sector therefore represents the highest geometric energy density allowed by the regularized spacetime structure.

Importantly, this sector is not directly observable at cosmological scales. The enormous separation between the Planck length and the cosmological causal scale suppresses its observable contribution.

The central objective of the present framework is to understand how this ultraviolet vacuum sector is projected into the observable infrared domain through the causal filtering mechanism developed in the next section.

3 Causal Filtering and the Observable Infrared Sector

3.1 Definition of the Causal Horizon

The present framework assumes a homogeneous and isotropic spacetime described at large scales by the FLRW geometry. The observable domain of a given observer is limited by the cosmological causal scale $R_H(t)$, defined as the characteristic horizon associated with the propagation of physical information.

This horizon does not represent a physical boundary of spacetime. Rather, it defines the maximum causally connected region accessible to observation at a given cosmological epoch.

3.2 Observable Domain and Horizon Scale

The ultraviolet sector of the theory is characterized by a universal curvature saturation scale

$$K_P = \frac{3}{l_P^2}, \tag{6}$$

where l_P denotes the Planck length.

At cosmological scales, however, physical observations are necessarily restricted to the finite causal domain characterized by the horizon scale $R_H(t)$.

The coexistence of these two scales naturally defines the dimensionless ratio

$$\frac{l_P}{R_H(t)}. \tag{7}$$

This ratio measures the separation between the fundamental ultraviolet scale and the macroscopic causal scale accessible to observation.

3.3 UV–IR Projection Law

Curvature possesses dimensions of inverse length squared. Therefore, the only dimensionless suppression factor that can be constructed from the scales l_P and $R_H(t)$ while preserving the dimensional structure of curvature is

$$\left(\frac{l_P}{R_H(t)}\right)^2. \tag{8}$$

We therefore introduce the effective infrared curvature associated with the observable causal domain:

$$K_{\text{eff}}(t) = K_P \left(\frac{l_P}{R_H(t)}\right)^2. \tag{9}$$

Using the ultraviolet saturation condition $K_P = 3/l_P^2$, the relation immediately reduces to

$$K_{\text{eff}}(t) = \frac{3}{R_H(t)^2}. \tag{10}$$

This expression establishes a direct connection between the ultraviolet curvature scale and the macroscopic causal structure of the observable Universe.

3.4 Effective Curvature Residue

Within this interpretation, the observable infrared vacuum sector corresponds to the residual curvature remaining after the ultraviolet sector is projected onto the finite causal domain accessible to the observer.

The UV–IR relation should therefore be interpreted as an effective geometric law describing the observable sector of spacetime. No assumptions are made regarding regions beyond the causal horizon, and the extension of the mechanism to super-horizon domains remains an open problem.

"The horizon does not constitute a physical boundary of the Universe. It only defines the causal domain available to a given observer."

4 Ontological Clarifications and Domain of Validity

4.1 Single Homogeneous and Isotropic Spacetime

The present framework does not postulate multiple spacetime sectors or disconnected geometrical domains. A single spacetime manifold is assumed throughout the entire construction, described at large scales by the standard homogeneous and isotropic FLRW geometry.

The ultraviolet and infrared sectors do not correspond to different spacetimes. Rather, they represent different curvature regimes of the same underlying geometric structure.

Consequently, the causal filtering mechanism should be interpreted as a projection between scales within a unique spacetime rather than as an interaction between distinct geometrical backgrounds.

4.2 Absence of Physical Boundaries

The cosmological horizon does not represent a physical boundary, edge, or topological limit of the Universe.

The existence of a finite causal scale arises solely from the finite propagation speed of physical information and the expansion history of spacetime.

Therefore, the horizon does not separate different universes or disconnected regions. The spacetime manifold is assumed to extend continuously beyond the observable domain.

In this sense, the causal horizon is an observational and causal concept rather than a physical surface embedded in spacetime.

4.3 Horizon as a Causal Scale

The characteristic scale $R_H(t)$ appearing in the filtering relation

$$K_{\text{eff}}(t) = K_P \left(\frac{l_P}{R_H(t)} \right)^2 \tag{11}$$

is interpreted as the largest causally accessible scale associated with a given observer.

Its role is not to modify the fundamental structure of spacetime but to define the region over which physical information can be exchanged and observationally reconstructed.

The filtering mechanism therefore acts only on degrees of freedom contained within the causally connected domain.

The horizon serves as an effective infrared regulator determined by the causal structure of the Universe itself.

4.4 Observable Versus Super-Horizon Domains

The UV–IR projection developed in this work is restricted to the observable sector of the Universe.

The effective curvature

$$K_{\text{eff}}(t) = K_P \left(\frac{l_P}{R_H(t)} \right)^2 \tag{12}$$

describes the residual curvature accessible inside the causal domain defined by $R_H(t)$.

No assumptions are made regarding the detailed behavior of regions located beyond the cosmological horizon. Although the same spacetime is assumed to continue beyond the observable domain, those regions are causally disconnected and cannot currently be probed observationally.

Consequently, extending the filtering mechanism to super-horizon scales would require additional physical assumptions and lies beyond the scope of the present work.

The present framework should therefore be interpreted as an effective description of the observable Universe, while the extension toward global cosmological scales remains an open problem for future investigation.

4.5 Horizon Scale and Domain of Applicability

The UV–IR projection law proposed in this work,

$$K_{\text{eff}}(t) = K_P \left(\frac{l_P}{R_H(t)} \right)^2, \tag{13}$$

requires the specification of the infrared scale $R_H(t)$ entering the filtering mechanism.

In an expanding universe, several horizon scales can be defined, including the particle horizon, the event horizon, and the Hubble horizon. These scales correspond to different notions of causal accessibility and generally do not coincide numerically.

The present framework adopts the Hubble horizon,

$$R_H(t) = \frac{c}{H(t)}, \tag{14}$$

as the characteristic infrared scale associated with the observable filtering process.

This choice is motivated by two considerations.

First, the Hubble horizon provides the natural cosmological length scale associated with the expansion rate of the Universe. As such, it defines a physically meaningful infrared scale for any mechanism relating microscopic and macroscopic geometric sectors.

Second, substituting $R_H = c/H$ into the UV–IR projection law immediately yields

$$K_{\text{eff}}(t) = \frac{3H^2(t)}{c^2}, \tag{15}$$

establishing a direct connection between the filtered curvature sector and the cosmological dynamics described by the modified Friedmann equations derived later in this work.

Alternative horizon definitions may also be considered. For example, replacing R_H by the particle horizon leads to residual curvatures of comparable cosmological magnitude, differing only by numerical factors of order unity. Such alternatives correspond to different physical interpretations of the infrared sector and are not investigated in the present analysis.

It is important to emphasize that the causal horizon does not represent a physical boundary of spacetime. The Universe is assumed to remain homogeneous and isotropic beyond the observable domain. The filtering mechanism merely identifies the finite causal region accessible to a given observer.

Accordingly, the UV–IR projection law should be interpreted as an effective geometric relation associated with the observable causal sector of spacetime rather than as a statement about the global structure of the entire Universe. The present framework therefore describes the observable infrared vacuum sector selected by causal accessibility.

Extending the filtering mechanism to super-horizon domains would require additional assumptions regarding physics beyond direct observational accessibility and remains outside the scope of the present work.

5 Covariant Localization of the Filtering Mechanism

5.1 Auxiliary Scalar Field η

The causal filtering mechanism introduced in the previous sections is naturally formulated in terms of large-scale geometric information accumulated along the spacetime history.

To represent this information in a covariant manner, we introduce a dimensionless auxiliary scalar field η defined through the inverse d'Alembertian operator acting on the Ricci scalar:

$$\eta = \Box^{-1} R. \tag{16}$$

Equivalently,

$$\Box \eta = R, \tag{17}$$

where

$$\Box \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu \tag{18}$$

denotes the covariant d'Alembertian.

The field η does not represent a new fundamental matter degree of freedom. Instead, it acts as a geometric bookkeeping field encoding the cumulative curvature history of the spacetime.

In this sense, η provides a local covariant representation of information that would otherwise be described through explicitly nonlocal geometric quantities.

5.2 Local Representation of the Nonlocal Sector

Directly working with inverse differential operators generally leads to a nonlocal formulation of the theory.

To obtain a local covariant representation, we introduce an additional auxiliary scalar field ξ, which enforces the constraint $\Box\eta = R$ through the variational principle.

The nonlocal sector can then be represented by the action

$$S_{\mathrm{NL}} = \int d^4x \sqrt{-g} \left[\xi(\Box \eta - R) - \mathcal{V}(\eta) \right]. \tag{19}$$

After integration by parts and neglecting boundary terms, this expression becomes

$$S_{\mathrm{NL}} = \int d^4x \sqrt{-g} \left[-g^{\mu\nu} \partial_\mu \xi \partial_\nu \eta - \xi R - \mathcal{V}(\eta) \right]. \tag{20}$$

The resulting formulation is manifestly local and generally covariant.

The fields η and ξ serve only to localize the geometric filtering sector and do not introduce independent matter sources.

5.3 Effective Geometric Action

The complete gravitational action is constructed by combining the Einstein–Hilbert sector with the localized filtering contribution:

$$S_{\mathrm{eff}} = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g}\, R + S_{\mathrm{NL}} + S_{\mathrm{matter}}. \tag{21}$$

Substituting the localized form of the nonlocal sector yields

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[\left(\frac{c^4}{16\pi G} - \xi \right) R - g^{\mu\nu} \partial_\mu \xi \partial_\nu \eta - \mathcal{V}(\eta) \right] + S_{\text{matter}}. \tag{22}$$

This action provides a covariant geometric framework in which ultraviolet curvature saturation and causal filtering can be represented within a local variational formulation.

The detailed field equations obtained from this action are derived in the next section.

6 Modified Covariant Field Equations

6.1 Metric Variation

The modified gravitational field equations follow from the variation of the effective action introduced in the previous section,

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[\left(\frac{c^4}{16\pi G} - \xi \right) R - g^{\mu\nu} \partial_\mu \xi \partial_\nu \eta - \mathcal{V}(\eta) \right] + S_{\text{matter}}. \tag{23}$$

Performing the variation with respect to the inverse metric $g^{\mu\nu}$, one obtains

$$\left(\frac{c^4}{16\pi G} - \xi \right) G_{\mu\nu} + \left(g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu \right) \xi = \frac{1}{2} T_{\mu\nu}^{\text{matter}} + \frac{1}{2} T_{\mu\nu}^{\text{vac}}, \tag{24}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ is the Einstein tensor and the additional contributions generated by the kinetic and potential terms of the localized filtering sector are collected into an effective vacuum tensor $T_{\mu\nu}^{\text{vac}}$.

Variation with respect to the auxiliary fields yields

$$\Box \eta = R, \tag{25}$$

and

$$\Box \xi = \frac{d\mathcal{V}}{d\eta}. \tag{26}$$

Together, these equations define the complete covariant dynamical system associated with the localized filtering mechanism.

6.2 Effective Vacuum Tensor

The geometric contributions generated by the auxiliary fields can be written in the form of an effective energy-momentum tensor,

$$T_{\mu\nu}^{\text{vac}} = \partial_\mu \xi \partial_\nu \eta + \partial_\nu \xi \partial_\mu \eta - g_{\mu\nu} \partial_\alpha \xi \partial^\alpha \eta - g_{\mu\nu} \mathcal{V}(\eta). \tag{27}$$

This tensor does not represent a new physical matter source. Rather, it encodes the geometric response associated with the localized representation of the filtering sector.

The modified field equations may therefore be written in an Einstein-like form,

$$G_{\mu\nu} = \frac{8\pi G}{c^4} \left[T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\text{vac}} \right] + \Delta G_{\mu\nu}, \tag{28}$$

where $\Delta G_{\mu\nu}$ contains the derivative terms generated by the nonminimal coupling of ξ to the Ricci scalar after transferring the coupling factor.

At cosmological scales, these geometric contributions behave effectively as a vacuum component whose observable density is determined by the causal filtering mechanism discussed in previous sections.

6.3 Bianchi Consistency

A fundamental consistency requirement of any generally covariant theory is the preservation of the contracted Bianchi identities,

$$\nabla_\mu G^{\mu\nu} = 0. \tag{29}$$

Because the modified field equations originate from a diffeomorphism-invariant action, the conservation law follows automatically.

Taking the covariant divergence of the field equations yields

$$\nabla_\mu \left(T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{vac}}\right) = 0. \tag{30}$$

Therefore, the effective vacuum sector does not violate local energy-momentum conservation.

Any apparent evolution of the infrared vacuum density is compensated by the dynamics of the auxiliary geometric fields, preserving the covariant structure of the theory.

This result guarantees that the causal filtering mechanism remains fully compatible with general covariance and does not require phenomenological source terms or ad hoc energy-exchange prescriptions.

The preservation of the Bianchi identities therefore provides an important consistency check of the localized formulation developed in this work.

7 Infrared Vacuum Sector

7.1 Effective Curvature Density

The causal filtering mechanism projects the ultraviolet curvature sector onto the finite observable domain defined by the cosmological causal scale $R_H(t)$.

The resulting infrared curvature is given by

$$K_{\text{eff}}(t) = K_P \left(\frac{l_P}{R_H(t)}\right)^2, \tag{31}$$

which, using the ultraviolet saturation condition $K_P = 3/l_P^2$, reduces to

$$K_{\text{eff}}(t) = \frac{3}{R_H(t)^2}. \tag{32}$$

The corresponding effective vacuum energy density follows directly from the Einstein curvature-density relation

$$\rho_{\text{vac}}(t) = \frac{c^2}{8\pi G} K_{\text{eff}}(t). \tag{33}$$

Therefore,

$$\rho_{\text{vac}}(t) = \frac{3c^2}{8\pi G R_H(t)^2}. \tag{34}$$

The observable vacuum sector thus appears as the residual geometric energy density associated with the finite causal domain accessible to observation.

7.2 Emergent Running-Vacuum Behaviour

The infrared vacuum sector is not introduced as an independent cosmological fluid.

Its evolution follows directly from the evolution of the causal scale $R_H(t)$.

If the causal scale evolves proportionally to the inverse Hubble expansion rate,

$$R_H(t) \propto \frac{c}{H(t)}, \tag{35}$$

then the effective curvature becomes

$$K_{\text{eff}}(t) \propto H^2(t), \tag{36}$$

leading to

$$\rho_{\text{vac}}(t) \propto H^2(t). \tag{37}$$

A running-vacuum behavior therefore emerges naturally from the geometric filtering mechanism without introducing phenomenological vacuum-running parameters.

The observed cosmological vacuum appears dynamically linked to the evolution of the causal structure of spacetime itself.

7.3 Relation with the UV–IR Projection Law

The infrared vacuum sector can be interpreted as the observable manifestation of the ultraviolet curvature saturation scale after projection through the causal horizon.

The enormous hierarchy between the Planck length and the cosmological horizon,

$$l_P \ll R_H(t), \tag{38}$$

produces a strong suppression of the ultraviolet vacuum contribution.

As a consequence,

$$K_{\text{eff}}(t) \ll K_P, \tag{39}$$

even though both quantities originate from the same underlying geometric sector.

Within this framework, the small observed vacuum density is not produced by fine tuning. Instead, it results from the large separation between the ultraviolet saturation scale and the macroscopic causal scale of the observable Universe.

The UV–IR projection law therefore provides a geometric connection between Planck-scale curvature and cosmological vacuum observables while preserving a single underlying spacetime structure.

8 Discussion

8.1 Interpretation of the Observable Vacuum

Within the present framework, the observed vacuum sector is not interpreted as a fundamental cosmological component independent of spacetime geometry.

Instead, the infrared vacuum emerges as the residual curvature accessible inside the finite causal domain associated with a given observer.

The effective vacuum density therefore reflects the projection of an underlying ultraviolet curvature sector onto the observable region of the Universe.

From this perspective, the smallness of the observed vacuum density is not attributed to a cancellation mechanism or fine tuning. Rather, it results from the enormous hierarchy between the Planck scale and the cosmological causal scale.

The observable vacuum is thus interpreted as a geometric infrared residue rather than as a fundamental constant of nature.

This interpretation naturally restricts the validity of the UV–IR projection law to the observable causal domain. The resulting vacuum sector should therefore be understood as an observable infrared residue selected by causal accessibility rather than as a property assigned to the entire Universe beyond all horizons.

8.2 Comparison with Running Vacuum Models

The present framework shares an important phenomenological feature with Running Vacuum Models (RVMs): both predict an effective vacuum density that evolves approximately as

$$\rho_{\mathrm{vac}}(t) \propto H^2(t). \tag{40}$$

However, the conceptual origin of this behavior differs substantially.

In standard RVM approaches, the running vacuum law is introduced through phenomenological arguments or renormalization-group inspired constructions involving vacuum-running parameters.

In contrast, the present framework attributes the infrared vacuum behavior to the geometric projection of a saturated ultraviolet curvature sector through a finite causal scale.

Consequently, the running behavior emerges from the filtering mechanism itself and not from an independently postulated vacuum evolution law.

8.3 Comparison with Quintessence

The present model also differs significantly from quintessence scenarios.

Quintessence models introduce a new dynamical scalar field together with an associated potential energy responsible for the late-time acceleration of the Universe.

In the framework developed here, no additional physical matter field is introduced.

The dynamical behavior of the infrared vacuum arises from the evolution of the causal scale and the corresponding geometric filtering process.

The auxiliary fields employed in the covariant formulation serve only as a mathematical localization of the nonlocal sector and are not interpreted as new observable particles or matter components.

Therefore, the model remains entirely geometric in character.

8.4 Limitations of the Present Framework

Several important limitations should be emphasized.

First, the UV–IR projection law

$$K_{\mathrm{eff}}(t) = K_P \left(\frac{l_P}{R_H(t)} \right)^2 \tag{41}$$

is interpreted in the present work as an effective relation describing the observable causal sector of the Universe.

The detailed derivation of this relation from a more fundamental microscopic theory remains an open problem.

Second, the present analysis is restricted to the causally accessible domain defined by the horizon scale $R_H(t)$.

No assumptions are made regarding the detailed behavior of spacetime beyond the cosmological horizon, where direct observational verification is presently impossible.

Third, the cosmological consequences of possible energy exchange between the residual vacuum sector and matter or radiation have not been investigated here.

Such processes, together with potential implications for structure formation and late-time cosmological evolution, are left for future work.

The present paper should therefore be regarded as a covariant geometric formulation of the causal filtering mechanism rather than as a complete cosmological model.

Future developments will investigate whether the UV–IR projection law can be derived from a more fundamental microscopic description and whether the filtering mechanism can be consistently extended beyond the observable horizon.

A further limitation concerns the choice of the infrared scale entering the filtering law. In the present work, the Hubble horizon is adopted because it provides a direct connection between the effective curvature sector and cosmological dynamics. Alternative horizon definitions may lead to comparable cosmological magnitudes but correspond to different physical interpretations. A systematic investigation of such alternatives is deferred to future work.

9 Conclusions

In this work, we have developed a covariant geometric framework connecting an ultraviolet curvature saturation scale with the observable infrared vacuum sector through a causal filtering mechanism.

The construction starts from the existence of a universal Planckian curvature bound,

$$K_P = \frac{3}{l_P^2}, \tag{42}$$

which replaces divergent ultraviolet behavior by a regularized geometric core of finite curvature.

The observable infrared sector emerges through the projection of this ultraviolet curvature onto the finite causal domain accessible to a cosmological observer. This leads naturally to the effective relation

$$K_{\text{eff}}(t) = K_P \left(\frac{l_P}{R_H(t)} \right)^2, \tag{43}$$

or equivalently

$$K_{\text{eff}}(t) = \frac{3}{R_H(t)^2}. \tag{44}$$

The resulting vacuum sector is therefore interpreted as an observable geometric residue associated with the causal structure of spacetime rather than as an independent fundamental cosmological component.

A covariant localization of the filtering mechanism was developed through auxiliary scalar fields, allowing the construction of a local geometric action and modified field equations consistent with general covariance.

The framework naturally generates an effective infrared vacuum density exhibiting a running-vacuum behavior of the form

$$\rho_{\text{vac}}(t) \propto H^2(t), \tag{45}$$

without introducing phenomenological vacuum-running parameters, quintessence fields, or additional matter sectors.

The present work should be regarded as a geometric effective description of the observable causal domain. The microscopic origin of the UV–IR projection law, together with its possible extension beyond the observable horizon, remains an open problem.

Future work will investigate the complete cosmological implications of the framework, including vacuum–radiation energy exchange, effective matter production from the residual vacuum sector, and possible extensions of the causal filtering mechanism to larger spacetime domains.

These developments may provide a deeper connection between Planck-scale geometry, cosmological vacuum energy, and the large-scale evolution of the Universe.

Finally, it is important to emphasize that the UV–IR projection law developed here is interpreted as an effective geometric relation associated with the observable causal sector of spacetime. The framework does not require physical boundaries, multiple spacetime regions, or additional matter fields. Instead, it provides a parameter-free connection between Planck-scale curvature saturation and the infrared vacuum sector accessible to observation. Extensions of the mechanism beyond the observable horizon remain open questions for future investigation.

References

[1] A. Einstein, "Die Feldgleichungen der Gravitation," Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin, 844–847 (1915).

[2] A. Friedmann, "Über die Krümmung des Raumes," Zeitschrift für Physik **10**, 377–386 (1922).

[3] G. Lemaître, "A homogeneous Universe of constant mass and increasing radius accounting for the radial velocity of extragalactic nebulae," Annales de la Société Scientifique de Bruxelles **47**, 49–59 (1927).

[4] L. Bianchi, "Sugli spazi a tre dimensions che ammettono un gruppo continuo di movimenti," Memorie di Matematica e di Fisica della Società Italiana delle Scienze, Serie Terza, **11**, 267–352 (1902).

[5] R. M. Wald, *General Relativity*, University of Chicago Press (1984).

[6] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press (1982).

[7] R. P. Woodard, "Nonlocal Models of Cosmic Acceleration," Foundations of Physics **44**, 213–233 (2014).

[8] S. Deser and R. P. Woodard, "Nonlocal Cosmology," Physical Review Letters **99**, 111301 (2007).

[9] M. Maggiore and M. Mancarella, "Nonlocal gravity and dark energy," Physical Review D **90**, 023005 (2014).

[10] L. Modesto, "Super-renormalizable Quantum Gravity," Physical Review D **86**, 044005 (2012).

[11] L. Modesto and L. Rachwał, "Nonlocal quantum gravity: A review," International Journal of Modern Physics D **26**, 1730020 (2017).

[12] V. P. Frolov, M. A. Markov and V. F. Mukhanov, "Black Holes as Possible Sources of Closed and Semiclosed Worlds," Physical Review D **41**, 383 (1990).

[13] J. M. Bardeen, "Non-singular General Relativistic Gravitational Collapse," Proceedings of GR5, Tbilisi (1968).

[14] J. Solà, "Cosmological constant and vacuum energy: old and new ideas," Journal of Physics: Conference Series **453**, 012015 (2013).

[15] J. Solà and A. Gómez-Valent, "The ΛCDM cosmology: From inflation to dark energy through running Λ," International Journal of Modern Physics D **24**, 1541003 (2015).

[16] P. J. E. Peebles and B. Ratra, "The Cosmological Constant and Dark Energy," Reviews of Modern Physics **75**, 559–606 (2003).

[17] M. Ostrogradsky, "Mémoires sur les équations différentielles relatives au problème des isopérimètres," Memoires de l'Académie Impériale des Sciences de Saint-Pétersbourg **6**, 385–517 (1850).

[18] W. C. F. Echenique, "Ultraviolet Curvature Saturation and Residual Infrared Vacuum," Zenodo (2026). DOI: 10.5281/zenodo.20501608.